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रचितः मानव धर्म प्रणेता

सद्गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

Subject : Mathematics

Topic : QUADRATIC EQUATIONS

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Quadratic Equation

1. Equation v/s Identity:

A quadratic equation is satisfied by exactly two values of ' x ' which may be real or imaginary. The equation, $ax^2 + bx + c = 0$ is:

★ a quadratic equation if	$a \neq 0$	Two Roots
★ a linear equation if	$a = 0, b \neq 0$	One Root
★ a contradiction if	$a = b = 0, c \neq 0$	No Root
★ an identity if	$a = b = c = 0$	Infinite Roots

If a quadratic equation is satisfied by three distinct values of ' x ', then it is an identity.

Solved Example # 1: (i) $3x^2 + 2x - 1 = 0$ is a quadratic equation here a = 3.
(ii) $(x + 1)^2 = x^2 + 2x + 1$ is an identity in x.

Solution.: Here highest power of x in the given relation is 2 and this relation is satisfied by three different values x= 0, x = 1 and x = – 1 and hence it is an identity because a polynomial equation of n^{th} degree cannot have more than n distinct roots.

2. Relation Between Roots & Co-efficients:

(i) The solutions of quadratic equation, $ax^2 + bx + c = 0$, $(a \neq 0)$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression, $b^2 - 4ac \equiv D$ is called discriminant of quadratic equation.

(ii) If α, β are the roots of quadratic equation, $ax^2 + bx + c = 0, a \neq 0$. Then:

(a) $\alpha + \beta = -\frac{b}{a}$ (b) $\alpha\beta = \frac{c}{a}$ (c) $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$

(iii) A quadratic equation whose roots are α & β, is $(x - \alpha)(x - \beta) = 0$ i.e.
x^2 – (sum of roots) x + (product of roots) = 0

Solved Example # 2: If α and β are the roots of $ax^2 + bx + c = 0$, find the equation whose roots are $\alpha+2$ and $\beta+2$.

Solution. Replacing x by x – 2 in the given equation, the required equation is
$a(x - 2)^2 + b(x - 2) + c = 0$ i.e., $ax^2 - (4a - b)x + (4a - 2b + c) = 0$.

Solved Example # 3 The coefficient of x in the quadratic equation $x^2 + px + q = 0$ was taken as 17 in place of 13, its roots were found to be – 2 and – 15. Find the roots of the original equation.

Solution. Here q = (– 2) × (– 15) = 30, correct value of p = 13. Hence original equation is
$x^2 + 13x + 30 = 0$ as $(x + 10)(x + 3) = 0$ $\therefore$ roots are – 10, – 3

Self Practice Problems : 1. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then find the quadratic equation whose roots are

(i) $2\alpha, 2\beta$ (ii) α^2, β^2 (iii) $\alpha + 1, \beta + 1$ (iv) $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}$ (v) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

2. If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, show that $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$.

Ans.(1) (i) $ax^2 + 2bx + 4c = 0$ (ii) $a^2x^2 + (2ac - b^2)x + c^2 = 0$
(iii) $ax^2 - (2a - b)x + a + c - b = 0$ (iv) $(a + b + c)x^2 - 2(a - c)x + a - b + c = 0$
(v) $ac\,x^2 - (b^2 - 2ac)x + ac = 0$

3. Nature of Roots:

Consider the quadratic equation, $ax^2 + bx + c = 0$ having α, β as its roots; $D \equiv b^2 - 4ac$

- $D = 0$: Roots are equal $\alpha = \beta = -b/2a$
- $D \neq 0$: Roots are unequal
  - $a, b, c \in R$ & $D > 0$: Roots are real
    - $a, b, c \in Q$ & D is a perfect square $\Rightarrow$ Roots are rational
      - $\downarrow$ $a = 1$, $b, c \in I$ & D is a perfect square $\Rightarrow$ Roots are integral.
    - $a, b, c \in Q$ & D is not a perfect square $\Rightarrow$ Roots are irrational i.e. $\alpha = p + \sqrt{q}$, $\beta = p - \sqrt{q}$
  - $a, b, c \in R$ & $D < 0$: Roots are imaginary $\alpha = p + iq$, $\beta = p - iq$

Solved Example # 4: For what values of m the equation $(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has equal roots.

Solution.
Given equation is $(1 + m)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ ....(i)
Let D be the discriminant of equation (i).
Roots of equation (i) will be equal if D = 0.
or, $4(1 + 3m)^2 - 4(1 + m)(1 + 8m) = 0$

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or, $4(1 + 9m^2 + 6m - 1 - 9m^2 - 8m^2) = 0$

or, $m^2 - 3m = 0$ or, $m(m - 3) = 0$ $\therefore$ $m = 0, 3$.

Solved Example # 5: Find all the integral values of a for which the quadratic equation $(x - a)(x - 10) + 1 = 0$ has integral roots.

Solution.: Here the equation is $x^2 - (a + 10)x + 10a + 1 = 0$. Since integral roots will always be rational it means D should be a perfect square.

From (i) $D = a^2 - 20a + 96$.

$\Rightarrow$ $D = (a - 10)^2 - 4$ $\Rightarrow$ $4 = (a - 10)^2 - D$

If D is a perfect square it means we want difference of two perfect square as 4 which is possible only when $(a - 10)^2 = 4$ and $D = 0$.

$\Rightarrow$ $(a - 10) = \pm 2$ $\Rightarrow$ $a = 12, 8$

Solved Example # 6: If the roots of the equation $(x - a)(x - b) - k = 0$ be c and d, then prove that the roots of the equation $(x - c)(x - d) + k = 0$, are a and b.

Solution. By given condition

$(x - a)(x - b) - k \equiv (x - c)(x - d)$ or $(x - c)(x - d) + k \equiv (x - a)(x - b)$

Above shows that the roots of $(x - c)(x - d) + k = 0$ are a and b.

Self Practice Problems :

3. Let $4x^2 - 4(\alpha - 2)x + \alpha - 2 = 0$ $(\alpha \in R)$ be a quadratic equation. Find the value of α for which

(i) Both roots are real and distinct. (ii) Both roots are equal.

(iii) Both roots are imaginary (iv) Both roots are opposite in sign.

(v) Both roots are equal in magnitude but opposite in sign.

4. Find the values of a, if $ax^2 - 4x + 9 = 0$ has integral roots.

5. If $P(x) = ax^2 + bx + c$, and $Q(x) = -ax^2 + dx + c$, $ac \neq 0$ then prove that $P(x) \cdot Q(x) = 0$ has atleast two real roots.

Ans. (1) (i) $(-\infty, 2) \cup (3, \infty)$ (ii) $\alpha \in \{2, 3\}$

(iii) $(2, 3)$ (iv) $(-\infty, 2)$ (v) ϕ

(2) $a = \frac{1}{3}, -\frac{1}{4}$

4. Common Roots:

Consider two quadratic equations, $a_1x^2 + b_1x + c_1 = 0$ & $a_2x^2 + b_2x + c_2 = 0$.

(i) If two quadratic equations have both roots common, then the equation are identical and their co-efficient are in proportion. i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(ii) If only one root is common, then the common root 'α' will be: $\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$

Hence the condition for one common root is:

$$a_1\left[\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right]^2 + b_1\left[\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right] + c_1 = 0$$

$$\equiv (c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$$

Note : If $f(x) = 0$ & $g(x) = 0$ are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of $h(x) = a\,f(x) + bg(x) = 0$.

Solved Example # 7: If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots, show that $b + q = \frac{ap}{2}$.

Solution. Given equations are : $x^2 - ax + b = 0$ and $x^2 - px + q = 0$.

Let α be the common root. Then roots of equation (2) will be α and α. Let β be the other root of equation (1). Thus roots of equation (1) are α, β and those of equation (2) are α, α.

Now $\alpha + \beta = a$ .... (iii)

$\alpha\beta = b$ .... (iv)

$2\alpha = p$ .... (v)

$\alpha^2 = q$ .... (vi)

L.H.S. $= b + q = \alpha\beta + \alpha^2 = \alpha(\alpha + \beta)$ .... (vii)

and R.H.S. $= \frac{ap}{2} = \frac{(\alpha+\beta)\,2\alpha}{2} = \alpha(\alpha + \beta)$ .... (viii)

from (7) and (8), L.H.S. = R.H.S.

Solved Example # 8: If $a, b, c \in R$ and equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 9 = 0$ have a common root, show that $a : b : c = 1 : 2 : 9$.

Solution. Given equations are : $x^2 + 2x + 9 = 0$ ....(i)

and $ax^2 + bx + c = 0$ ....(ii)

Clearly roots of equation (i) are imaginary since equation (i) and (ii) have a common root, therefore common root must be imaginary and hence both roots will be common.

Therefore equations (i) and (ii) are identical

$\therefore$ $\frac{a}{1} = \frac{b}{2} = \frac{c}{9}$ $\therefore$ $a : b : c = 1 : 2 : 9$

Self Practice Problems : **6.** If the equation $x^2 + bx + ac = 0$ and $x^2 + cx + ab = 0$ have a common root then prove that the equation containing other roots will be given by $x^2 + ax + bc = 0$.

7. If the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two common roots then show that $a = b = c$.

8. If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P. show that a_1, b_1, c_1 are in G.P.

5. Factorisation of Quadratic Expressions:

★ The condition that a quadratic expression $f(x) = ax^2 + bx + c$ a perfect square of a linear expression, is $D \equiv b^2 - 4ac = 0$.

★ The condition that a quadratic expression $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factors is that;

$$\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \quad \text{OR} \quad \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

Solved Example # 9: Determine a such that $x^2 - 11x + a$ and $x^2 - 14x + 2a$ may have a common factor.

Solution. Let $x - \alpha$ be a common factor of $x^2 - 11x + a$ and $x^2 - 14x + 2a$.

Then $x = \alpha$ will satisfy the equations $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$.

$\therefore$ $\alpha^2 - 11\alpha + a = 0$ and $\alpha^2 - 14\alpha + 2a = 0$

Solving (i) and (ii) by cross multiplication method, we get a = 24.

Sol. Ex. 10: Show that the expression $x^2 + 2(a + b + c)x + 3(bc + ca + ab)$ will be a perfect square if a = b = c.

Solution. Given quadratic expression will be a perfect square if the discriminant of its corresponding equation is zero.

i.e. $4(a + b + c)^2 - 4.3\,(bc + ca + ab) = 0$

or $(a + b + c)^2 - 3(bc + ca + ab) = 0$

or $\frac{1}{2}\left((a - b)^2 + (b - c)^2 + (c - a)^2\right) = 0$

which is possible only when a = b = c.

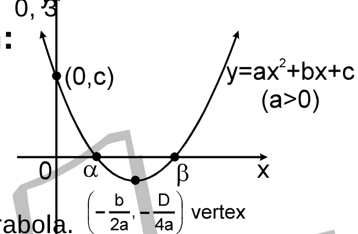
Self Practice Problems :

9. For what values of k the expression $(4 - k)x^2 + 2(k + 2)x + 8k + 1$ will be a perfect square ?

10. If $x - \alpha$ be a factor common to $a_1x^2 + b_1x + c$ and $a_2x^2 + b_2x + c$ prove that $\alpha(a_1 - a_2) = b_2 - b_1$.

11. If $3x^2 + 2\alpha xy + 2y^2 + 2ax - 4y + 1$ can be resolved into two linear factors, Prove that α is a root of the equation $x^2 + 4ax + 2a^2 + 6 = 0$. **Ans.** (1) 0, 3

6. Graph of Quadratic Expression:



$y = f(x) = ax^2 + bx + c$

or $\left(y + \frac{D}{4a}\right) = a\left(x + \frac{b}{2a}\right)^2$

★ the graph between x, y is always a parabola.

★ the co–ordinate of vertex are $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

★ If a > 0 then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.

★ the parabola intersect the y–axis at point (0, c).

★ the x–co–ordinate of point of intersection of parabola with x–axis are the real roots of the quadratic equation f (x) = 0. Hence the parabola may or may not intersect the x–axis at real points.

7. Range of Quadratic Expression $f(x) = ax^2 + bx + c$.

(i) Absolute Range:

If $a > 0 \Rightarrow f(x) \in \left[-\frac{D}{4a}, \infty\right)$

$a < 0 \Rightarrow f(x) \in \left(-\infty, -\frac{D}{4a}\right]$

Hence maximum and minimum values of the expression f (x) is $-\frac{D}{4a}$ in respective cases and it occurs at $x = -\frac{b}{2a}$ (at vertex).

(ii) Range in restricted domain: Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$$f(x) \in \left[\min\{f(x_1), f(x_2)\}, \quad \max\{f(x_1), f(x_2)\}\right]$$

(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

$$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}, \quad \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$$

Solved Example # 11 If c < 0 and $ax^2 + bx + c = 0$ does not have any real roots then prove that

(i) $a - b + c < 0$ (ii) $9a + 3b + c < 0$.

Solution.

$c < 0$ and $D < 0$ $\Rightarrow$ $f(x) = ax^2 + bx + c < 0$ for all $x \in R$

$\Rightarrow$ $f(-1) = a - b + c < 0$

and $f(3) = 9a + 3b + c < 0$

Solved Example # 12 Find the maximum and minimum values of $f(x) = x^2 - 5x + 6$.

Solution.

minimum of $f(x) = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$

$= -\left(\frac{25-24}{4}\right)$ at $x = \frac{5}{2}$ $\quad = -\frac{1}{4}$

maximum of $f(x) = \infty$ Hence range is $\left[-\frac{1}{4}, \infty\right)$.

Solved Example # 13 : Find the range of rational expression $y = \frac{x^2 - x + 1}{x^2 + x + 1}$ if x is real.

Solution. $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow$ $(y - 1)x^2 + (y + 1)x + y - 1 = 0$

$\therefore$ x is real $\therefore$ $D \geq 0$

$\Rightarrow$ $(y + 1)^2 - 4(y - 1)^2 \geq 0$ $\Rightarrow$ $(y - 3)(3y - 1) \leq 0$ $\Rightarrow$ $y \in \left[\frac{1}{3}, 3\right]$.

Solved Example # 14: Find the range of $y = \frac{x + 2}{2x^2 + 3x + 6}$, if x is real.

Solution.: $y = \frac{x + 2}{2x^2 + 3x + 6}$

$\Rightarrow$ $2yx^2 + 3yx + 6y = x + 2$ $\Rightarrow$ $2yx^2 + (3y - 1)x + 6y - 2 = 0$

$\therefore$ x is real

$D \geq 0$

$\Rightarrow$ $(3y - 1)^2 - 8y(6y - 2) \geq 0$ $\Rightarrow$ $(3y - 1)(13y + 1) \leq 0$

$y \in \left[-\frac{1}{13}, \frac{1}{3}\right]$.

Self Practice Problems :

12. If $c > 0$ and $ax^2 + 2bx + 3c = 0$ does not have any real roots then prove that
(i) $a - 2b + 3c > 0$ (ii) $a + 4b + 12c > 0$

13. If $f(x) = (x - a)(x - b)$, then show that $f(x) \geq -\frac{(a-b)^2}{4}$.

14. For what least integral value of k the quadratic polynomial $(k - 2)x^2 + 8x + k + 4 > 0 \ \forall x \in R$.

15. Find the range in which the value of function $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ lies $\forall x \in R$.

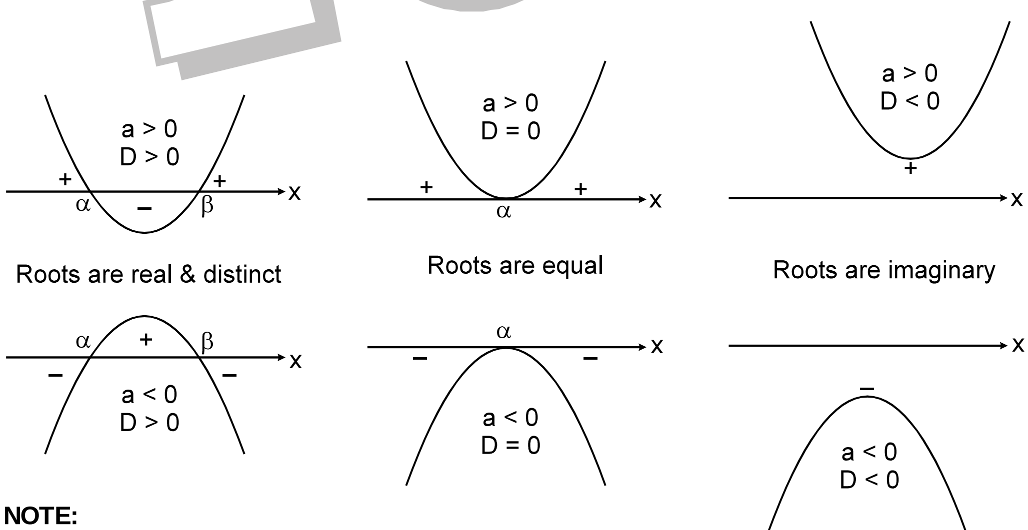
16. Find the interval in which 'm' lies so that the function $y = \frac{mx^2 + 3x - 4}{-4x^2 + 3x + m}$ can take all real values $\forall x \in R$.

Ans. (14) k = 5. (15) $(-\infty, 5] \cup [9, \infty)$ (16) $m \in [1, 7]$

8. Sign of Quadratic Expressions:

The value of expression, $f(x) = ax^2 + bx + c$ at $x = x_0$ is equal to y–co–ordinate of a point on parabola $y = ax^2 + bx + c$ whose x–co–ordinate is x_0. Hence if the point lies above the x–axis for some $x = x_0$, then $f(x_0) > 0$ and vice–versa.

We get six different positions of the graph with respect to x–axis as shown.



NOTE:

(i) $\forall x \in R$, $y > 0$ only if $a > 0$ & $D \equiv b^2 - 4ac < 0$ (figure 3).

(ii) $\forall x \in R$, $y < 0$ only if $a < 0$ & $D \equiv b^2 - 4ac < 0$ (figure 6).

9. Solution of Quadratic Inequalities:

The values of 'x' satisfying the inequality, $ax^2 + bx + c > 0$ $(a \neq 0)$ are:

(i) If $D > 0$, i.e. the equation $ax^2 + bx + c = 0$ has two different roots $\alpha < \beta$.
Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\beta, \infty)$

$a < 0 \Rightarrow x \in (\alpha, \beta)$

(ii) If D = 0, i.e. roots are equal, i.e. $\alpha = \beta$.
Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$
$a < 0 \Rightarrow x \in \phi$

(iii) If D < 0, i.e. the equation $ax^2 + bx + c = 0$ has no real root.
Then $a > 0 \Rightarrow x \in R$
$a < 0 \Rightarrow x \in \phi$

(iv) Inequalities of the form $\frac{P(x)\ Q(x)\ R(x).........}{A(x)\ B(x)\ C(x).........} \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ can be quickly solved using the method of intervals, where A, B, C........, P, Q, R......... are linear functions of ' x '.

Solved Example # 15 Solve $\frac{x^2 + 6x - 7}{x^2 + 1} \le 2$

Solution. $\Rightarrow$ $x^2 + 6x - 7 \le 2x^2 + 2$
$\Rightarrow$ $x^2 - 6x + 9 \ge 0$ $\Rightarrow$ $(x - 3)^2 \ge 0$ $\Rightarrow$ $x \in R$

Solved Example # 16: Solve $\frac{x^2 + x + 1}{|x + 1|} > 0.$

Solution. $\therefore$ $|x + 1| > 0$
$\forall x \in R - \{-1\}$
$\therefore$ $x^2 + x + 1 > 0$ $\therefore$ $D = 1 - 4 = -3 < 0$
$\therefore$ $x^2 + x + 1 > 0 \ \forall x \in R$ $\therefore$ $x \in (-\infty, -1) \cup (-1, \infty)$

Solved Example # 17 $\left|\frac{x^2 - 3x - 1}{x^2 + x + 1}\right| < 3.$

Solution. $\frac{|x^2 - 3x - 1|}{x^2 + x + 1} < 3.$
$\because$ in $x^2 + x + 1$
$D = 1 - 4 = -3 < 0$
$\therefore$ $x^2 + x + 1 > 0 \ \forall x \in R$ $\therefore$ $|x^2 - 3x - 1| < 3(x^2 + x + 1)$
$\Rightarrow$ $(x^2 - 3x - 1)^2 - \{3(x^2 + x + 1)\}^2 < 0$
$\Rightarrow$ $(4x^2 + 2)(-2x^2 - 6x - 4) < 0$
$\Rightarrow$ $(2x^2 + 1)(x + 2)(x + 1) > 0$ $\Rightarrow$ $x \in (-\infty, -2) \cup (-1, \infty)$

Self Practice Problems :

17. (i) $|x^2 + x| - 5 < 0$ (ii) $x^2 - 7x + 12 < |x - 4|$

18. Solve $\frac{2x}{x^2 - 9} \le \frac{1}{x + 2}$

19. Solve the inequation $(x^2 + 3x + 1)(x^2 + 3x - 3) \ge 5$

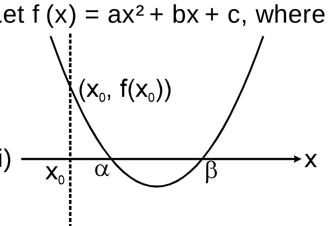
20. Find the value of parameter 'α' for which the inequality $\left|\frac{x^2 + \alpha x + 1}{x^2 + x + 1}\right| < 3$ is satisfied $\forall x \in R$

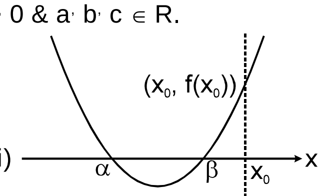
21. Solve $\left|\frac{x^2 - 5x + 4}{x^2 - 4}\right| \le 1$

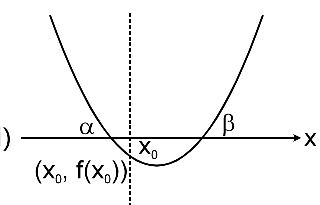
Ans. (17) (i) $\left(-\left(\frac{1+\sqrt{21}}{2}\right), \left(\frac{\sqrt{21}-1}{2}\right)\right)$ (ii) (2, 4)
(18) $(-\infty, -3) \cup (-2, 3)$ (19) $(-\infty, -4] \cup [-2, -1] \cup [1, \infty)$
(20) (–1, 5) (21) $\left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$

10. Location Of Roots:

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a \cdot b \cdot c \in R$.

(i) 

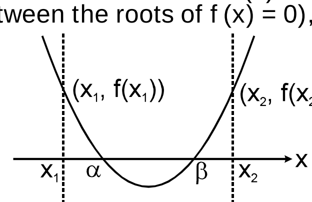
(ii) 

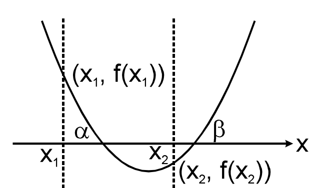
(iii) 

(i) Conditions for both the roots of f (x) = 0 to be greater than a specified number 'x_0' are $b^2 - 4ac \ge 0$; $f(x_0) > 0$ & $(-b/2a) > x_0$.

(ii) Conditions for both the roots of f (x) = 0 to be smaller than a specified number 'x_0' are $b^2 - 4ac \ge 0$; $f(x_0) > 0$ & $(-b/2a) < x_0$.

(iii) Conditions for both roots of f (x) = 0 to lie on either side of the number 'x_0' (in other words the number 'x_0' lies between the roots of f (x) = 0), is $f(x_0) < 0$.

(iv) 

(v) 

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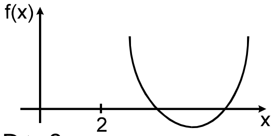
(iv) Conditions that both roots of f (x) = 0 to be confined between the numbers x_1 and x_2, $(x_1 < x_2)$ are $b^2 - 4ac \geq 0$; $f(x_1) > 0$; $f(x_2) > 0$ & $x_1 < (-b/2a) < x_2$.

(v) Conditions for exactly one root of f (x) = 0 to lie in the interval (x_1, x_2) i.e. $x_1 < x < x_2$ is $f(x_1) . f(x_2) < 0$.

Ex.10.1 $x^2 - (m - 3)x + m = 0$

(a) Find values of m so that both the roots are greater than 2.



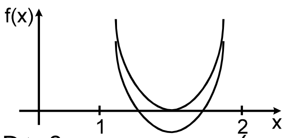
Condition - f $D \geq 0$ $\Rightarrow$ $(m - 3)^2 - 4m \geq 0$ $\Rightarrow$ $m^2 - 10m + 9 \geq 0$
$\Rightarrow$ $(m - 1)(m - 9) \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$ ......(i)

Condition - ff $f(2) > 0$ $\Rightarrow$ $4 - (m - 3)2 + m > 0$ $\Rightarrow$ $m < 10$...(ii),

Condition - fff $-\dfrac{b}{2a} > 2$ $\Rightarrow$ $\dfrac{m-3}{2} > 2$ $\Rightarrow$ $m > 7$.....(iii)

Intersection of (i), (ii) and (iii) gives $m \in [9, 10)$ **Ans.**

(b) Find the values of m so that both roots lie in the interval (1, 2)

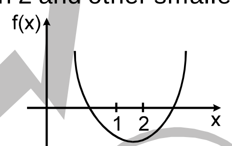


Condition - f $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$
Condition - ff $f(1) > 0$ $\Rightarrow$ $1 - (m - 3) + m > 0$ $\Rightarrow$ $4 > 0$ $\Rightarrow$ $m \in R$
Condition - fff $f(2) > 0$ $\Rightarrow$ $m < 10$

Condition - fV $1 < -\dfrac{b}{2a} < 2$ $\Rightarrow$ $1 < \dfrac{m-3}{2} < 2$ $\Rightarrow$ $5 < m < 7$

intersection gives $m \in \phi$ **Ans.**

(c) One root is greater than 2 and other smaller than 1



Condition - f $f(1) < 0$ $\Rightarrow$ $4 < 0$ $\Rightarrow$ $m \in \phi$
Condition - ff $f(2) < 0$ $\Rightarrow$ $m > 10$
Intersection gives $m \in \phi$ **Ans.**

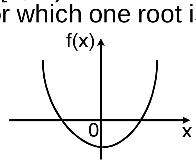
(d) Find the value of m for which both roots are positive.

Condition - f $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$
Condition - ff $f(0) > 0$ $\Rightarrow$ $m > 0$

Condition - fff $-\dfrac{b}{2a} > 0$ $\Rightarrow$ $\dfrac{m-3}{2} > 0$ $\Rightarrow$ $m > 3$

intersection gives $m \in [9, \infty)$ **Ans.**

(e) Find the values of m for which one root is (positive) and other is (negative).

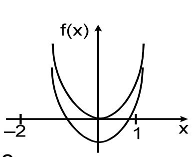


Condition - f $f(0) < 0$ $\Rightarrow$ $m < 0$ **Ans.**

(f) Roots are equal in magnitude and opposite in sign.
sum of roots = 0 $\Rightarrow$ $m = 3$
and $f(0) < 0$ $\Rightarrow$ $m < 0$
$\therefore$ $m \in \phi$ **Ans.**

Ex.10.2 Find all the values of 'a' for which both the roots of the equation $(a - 2)x^2 + 2ax + (a + 3) = 0$ lies in the interval (– 2, 1).

Sol. **Case - f**



When $a - 2 > 0$
$\Rightarrow$ $a > 2$

Condition - f $f(-2) > 0$ $\Rightarrow$ $(a - 2)4 - 4a + a + 3 > 0$ $\Rightarrow$ $a - 5 > 0$ $\Rightarrow$ $a > 5$

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Condition - ii $f(1) > 0 \Rightarrow 4a + 1 > 0 \Rightarrow a > -\frac{1}{4}$

Condition - iii $D \geq 0 \Rightarrow 4a^2 - 4(a + 3)(a - 2) \geq 0 \Rightarrow a \leq 6$

Condition - iv $-\frac{b}{2a} < 1 \Rightarrow \frac{2(a-1)}{a-2} > 0 \Rightarrow a \in (-\infty, 1) \cup (4, \infty)$

Condition - V $-2 < -\frac{b}{2a} \Rightarrow \frac{-2a}{2(a-2)} > -2 \Rightarrow \frac{a-4}{a-2} > 0$

Intersection gives $a \in (5, 6]$. **Ans.**

Case-ii when $a - 2 < 0$
$a < 2$

Condition - i $f(-2) < 0 \Rightarrow a < 5$

Condition - ii $f(1) < 0, \Rightarrow a < -\frac{1}{4}$

Condition - iii $-2 < -\frac{b}{2a} < 1 \Rightarrow a \in (-\infty, 1) \cup (4, \infty)$

Condition - iV $D \geq 0 \Rightarrow a \leq 6$

intersection gives $a \in \left(-\infty, -\frac{1}{4}\right)$

complete solution is $a \in \left(-\infty, -\frac{1}{4}\right) \cup (5, 6]$ **Ans.**

Self Practice Problems :

22. Let $4x^2 - 4(\alpha - 2)x + \alpha - 2 = 0$ ($\alpha \in R$) be a quadratic equation find the value of α for which
(a) Both the roots are positive (b) Both the roots are negative
(c) Both the roots are opposite in sign. (d) Both the roots are greater than 1/2.
(e) Both the roots are smaller than 1/2.
(f) One root is small than 1/2 and the other root is greater than 1/2.
Ans. (a) $[3, \infty)$ (b) ϕ (c) $(-\infty, 2)$ (d) ϕ (e) $(-\infty, 2]$ (f) $(3, \infty)$

23. Find the values of the parameter a for which the roots of the quadratic equation $x^2 + 2(a - 1)x + a + 5 = 0$ are
(i) positive (ii) negative (iii) opposite in sign.
Ans. (i) $(-5, -1]$ (ii) $[4, \infty)$ (iii) $(-\infty, -5)$

24. Find the values of P for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$ are less than 2.
Ans. $(-\infty, -1)$

25. Find the values of α for which 6 lies between the roots of the equation $x^2 + 2(\alpha - 3)x + 9 = 0$.
Ans. $\left(-\infty, -\frac{3}{4}\right)$.

26. Let $4x^2 - 4(\alpha - 2)x + \alpha - 2 = 0$ ($\alpha \in R$) be a quadratic equation find the value of α for which
(i) Exactly one root lies in $\left(0, \frac{1}{2}\right)$. (ii) Both roots lies in $\left(0, \frac{1}{2}\right)$.
(iii) At least one root lies in $\left(0, \frac{1}{2}\right)$. (iv) One root is greater than 1/2 and other root is smaller than 0.
Ans. (i) $(-\infty, 2) \cup (3, \infty)$ (ii) ϕ (iii) $(-\infty, 2) \cup (3, \infty)$ (iv) ϕ

27. In what interval must the number 'a' vary so that both roots of the equation $x^2 - 2ax + a^2 - 1 = 0$ lies between -2 and 4. **Ans.** $(-1, 3)$

28. Find the values of k, for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x. **Ans.** $[1, 2)$

11. Theory Of Equations:

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ... a_n$ are all real & $a_0 \neq 0$ then,

$\sum \alpha_1 = -\frac{a_1}{a_0}$, $\sum \alpha_1\alpha_2 = +\frac{a_2}{a_0}$, $\sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$, ..., $\alpha_1\alpha_2\alpha_3.......\alpha_n = (-1)^n \frac{a_n}{a_0}$

NOTE :

(i) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of n^{th} degree ($n \geq 1$) has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. imaginary roots occur in conjugate pairs.

(iv) An equation of odd degree will have odd number of real roots and an equation of even degree will have even numbers of real roots.

(v) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.

(vi) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have odd number of real roots (also atleast one real root) between 'a' and 'b'.

(vii) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term. (If coefficient of highest degree term is positive).

Ex.11.1 $2x^3 + 3x^2 + 5x + 6 = 0$ has roots α, β, γ then find $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.

$\therefore \quad \alpha + \beta + \gamma = = -\dfrac{3}{2} \qquad \alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{5}{2}, \qquad \alpha\beta\gamma = -\dfrac{6}{2} = -3.$

Ex.11.2 Find the roots of $4x^3 + 20x^2 - 23x + 6 = 0$. If two roots are equal.

Let roots be α, α and β

$\therefore \quad \alpha + \alpha + \beta = -\dfrac{20}{4}$

$\Rightarrow \quad 2\alpha + \beta = -5 \qquad$ ....(i)

$\therefore \quad \alpha . \alpha + \alpha\beta + \alpha\beta = -\dfrac{23}{4}$

$\Rightarrow \quad \alpha^2 + 2\alpha\beta = -\dfrac{23}{4} \qquad \& \qquad \alpha^2\beta = -\dfrac{6}{4}$

from equation (i)

$\alpha^2 + 2\alpha(-5 - 2\alpha) = -\dfrac{23}{4}$

$\Rightarrow \quad \alpha^2 - 10\alpha - 4\alpha^2 = -\dfrac{23}{4} \quad \Rightarrow \quad 12\alpha^2 + 40\alpha - 23 = 0$

$\therefore \quad \alpha = 1/2, -\dfrac{23}{6} \qquad$ when $\alpha = \dfrac{1}{2}$

from equation (i) $\qquad \alpha^2\beta = \dfrac{1}{4}(-5 - 1) = -\dfrac{3}{2}$

when $\alpha = -\dfrac{23}{6}$

$\alpha^2\beta = \dfrac{23 \times 23}{36}\left(-5 - 2x\left(-\dfrac{23}{6}\right)\right) \neq -\dfrac{3}{2}$

$\Rightarrow \quad \alpha = \dfrac{1}{2}, \qquad \beta = -6$

Hence roots of equation $\qquad = \dfrac{1}{2}, \dfrac{1}{2}, -6$ **Ans.**

Self Practice Problems :

29. Find the relation between p, q and r if the roots of the cubic equation $x^3 - px^2 + qx - r = 0$ are such that they are in A.P. **Ans.** $2p^3 - 9pq + 27r = 0$

30. If α, β, γ are the roots of the cubic $x^3 + qx + r = 0$ then find the equation whose roots are

(a) $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ **Ans.** $x^3 + qx - r = 0$
(b) $\alpha\beta, \beta\gamma, \gamma\alpha$ **Ans.** $x^3 - qx^2 - r^2 = 0$
(c) $\alpha^2, \beta^2, \gamma^2$ **Ans.** $x^3 + 2qx^2 + q^2x - r^2 = 0$
(d) $\alpha^3, \beta^3, \gamma^3$ **Ans.** $x^3 + 3x^2r + (3r^2 + q^3)x + r^3 = 0$

SHORT REVISION

The general form of a quadratic equation in x is, $ax^2 + bx + c = 0$, where $a, b, c \in R \; \& \; a \neq 0$.

RESULTS :

1. The solution of the quadratic equation, $ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $b^2 - 4ac = D$ is called the discriminant of the quadratic equation.

2. If $\alpha \; \& \; \beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then;

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$ (iii) $\alpha - \beta = \sqrt{D}/a$.

3. **NATURE OF ROOTS:**

(A) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in R \; \& \; a \neq 0$ then ;

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).
(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).
(iii) $D < 0 \Leftrightarrow$ roots are imaginary .
(iv) If $p + iq$ is one root of a quadratic equation, then the other must be the conjugate $p - iq$ & vice versa. $(p, q \in R \; \& \; i = \sqrt{-1})$.

(B) Consider the quadratic equation $ax^2 + bx + c = 0$ where $a, b, c \in Q \; \& \; a \neq 0$ then;

(i) If $D > 0$ & is a perfect square, then roots are rational & unequal.
(ii) If $\alpha = p + \sqrt{q}$ is one root in this case, (where p is rational & $\sqrt{q}$ is a surd) then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ & vice versa.

4. A quadratic equation whose roots are $\alpha \; \& \; \beta$ is $(x - \alpha)(x - \beta) = 0$ i.e. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e. $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$.

5. Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

6. **Consider the quadratic expression,** $y = ax^2 + bx + c$, $a \neq 0 \; \& \; a, b, c \in R$ then ;

(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $\forall\ x \in R,\ y > 0$ only if $a > 0$ & $b^2 - 4ac < 0$ (figure 3).
(iii) $\forall\ x \in R,\ y < 0$ only if $a < 0$ & $b^2 - 4ac < 0$ (figure 6).

Carefully go through the 6 different shapes of the parabola given below.

7. SOLUTION OF QUADRATIC INEQUALITIES:

$ax^2 + bx + c > 0\ (a \neq 0)$.

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $x_1 < x_2$.
Then $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_2, \infty)$
$a < 0 \Rightarrow x \in (x_1, x_2)$

(ii) If $D = 0$, then roots are equal, i.e. $x_1 = x_2$.
In that case $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_1, \infty)$
$a < 0 \Rightarrow x \in \phi$

(iii) Inequalities of the form $\frac{P(x)}{Q(x)}$ 0 can be quickly solved using the method of intervals.

8. MAXIMUM & MINIMUM VALUE of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as;

$a < 0$ or $a > 0$. $y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$ if $a > 0$ & $y \in \left(-\infty, \frac{4ac - b^2}{4a}\right]$ if $a < 0$.

9. COMMON ROOTS OF 2 QUADRATIC EQUATIONS [ONLY ONE COMMON ROOT]:

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$. Therefore

$a\alpha^2 + b\alpha + c = 0$; $a'\alpha^2 + b'\alpha + c' = 0$. By Cramer's Rule $\frac{\alpha^2}{bc' - b'c} = \frac{\alpha}{a'c - ac'} = \frac{1}{ab' - a'b}$

Therefore, $\alpha = \frac{ca' - c'a}{ab' - a'b} = \frac{bc' - b'c}{a'c - ac'}$.

So the condition for a common root is $(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$.

10. The condition that a quadratic function $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ may be resolved into two linear factors is that;

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ OR $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

11. THEORY OF EQUATIONS:

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ... a_n$ are all real & $a_0 \neq 0$ then,

$\sum\alpha_1 = -\frac{a_1}{a_0}$, $\sum\alpha_1\alpha_2 = +\frac{a_2}{a_0}$, $\sum\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$, ..., $\alpha_1\alpha_2\alpha_3........\alpha_n = (-1)^n\frac{a_n}{a_0}$

Note:
(i) If α is a root of the equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of $f(x)$ and conversely.
(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.
(iii) If the coefficients of the equation $f(x) = 0$ are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. **imaginary roots occur in conjugate pairs**.
(iv) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not a perfect square.
(v) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b'.
(vi) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

12. LOCATION OF ROOTS: Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'd' are. $b^2 - 4ac \geq 0$; $f(d) > 0$ & $(-b/2a) > d$.
(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'd' (in other words the number 'd' lies between the roots of $f(x) = 0$) is $f(d) < 0$.
(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval (d, e) i.e. $d < x < e$ are $b^2 - 4ac > 0$ & $f(d) \cdot f(e) < 0$.
(iv) Conditions that both roots of $f(x) = 0$ to be confined between the numbers p & q are $(p < q)$. $b^2 - 4ac \geq 0$; $f(p) > 0$; $f(q) > 0$ & $p < (-b/2a) < q$.

13. LOGARITHMIC INEQUALITIES

(i) For $a > 1$ the inequality $0 < x < y$ & $\log_a x < \log_a y$ are equivalent.
(ii) For $0 < a < 1$ the inequality $0 < x < y$ & $\log_a x > \log_a y$ are equivalent.
(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$
(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$
(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$
(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

EXERCISE–1

Q.1 If the roots of the equation $[1/(x + p)] + [1/(x + q)] = 1/r$ are equal in magnitude but opposite in sign, show that $p + q = 2r$ & that the product of the roots is equal to $(-1/2)(p^2 + q^2)$.

Q.2 If $x^2 - x\cos(A + B) + 1$ is a factor of the expression,
$2x^4 + 4x^3\sin A\sin B - x^2(\cos 2A + \cos 2B) + 4x\cos A\cos B - 2$. Then find the other factor.

Q.3 α, β are the roots of the equation $K(x^2 - x) + x + 5 = 0$. If K_1 & K_2 are the two values of K for which the roots α, β are connected by the relation $(\alpha/\beta) + (\beta/\alpha) = 4/5$. Find the value of $(K_1/K_2) + (K_2/K_1)$.

Q.4 If the quadratic equations, $x^2 + bx + c = 0$ and $bx^2 + cx + 1 = 0$ have a common root then prove that either $b + c + 1 = 0$ or $b^2 + c^2 + 1 = bc + b + c$.

Q.5 If the roots of the equation $\left(1 - q + \frac{p^2}{2}\right)x^2 + p(1 + q)x + q(q - 1) + \frac{p^2}{2} = 0$ are equal then show that

$p^2 = 4q$.

Q.6 If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$.

Q.7 Find the range of values of a, such that $f(x) = \dfrac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

Q.8 Find a quadratic equation whose sum and product of the roots are the values of the expressions (cosec 10° – $\sqrt{3}$ sec10°) and (0.5 cosec10° – 2 sin70°) respectively. Also express the roots of this quadratic in terms of tangent of an angle lying in $\left(0, \dfrac{\pi}{2}\right)$.

Q.9 Find the least value of $\dfrac{6x^2 - 22x + 21}{5x^2 - 18x + 17}$ for all real values of x, using the theory of quadratic equations.

Q.10 Find the least value of $(2p^2 + 1)x^2 + 2(4p^2 - 1)x + 4(2p^2 + 1)$ for real values of p and x.

Q.11 If α be a root of the equation $4x^2 + 2x - 1 = 0$ then prove that $4\alpha^3 - 3\alpha$ is the other root.

Q.12(a) If α, β are the roots of the quadratic equation $ax^2+bx+c = 0$ then which of the following expressions in α, β will denote the symmetric functions of roots. Give proper reasoning. (i) $f(\alpha, \beta) = \alpha^2 - \beta$ (ii) $f(\alpha, \beta) = \alpha^2\beta + \alpha\beta^2$ (iii) $f(\alpha, \beta) = \ln \dfrac{\alpha}{\beta}$ (iv) $f(\alpha, \beta) = \cos(\alpha - \beta)$

(b) If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ & $\alpha^3\beta^2 + \alpha^2\beta^3$.

Q.13 If α, β are the roots of $ax^2 + bx + c = 0$ & $\alpha', -\beta$ are the roots of $a'x^2 + b'x + c' = 0$, show that α, α' are the roots of $\left[\dfrac{b}{a} + \dfrac{b'}{a'}\right]^{-1} x^2 + x + \left[\dfrac{b}{c} + \dfrac{b'}{c'}\right]^{-1} = 0$.

Q.14 If α, β are the roots of $x^2 - px + 1 = 0$ & γ, δ are the roots of $x^2 + qx + 1 = 0$, show that $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = q^2 - p^2$.

Q.15 Show that if p, q, r & s are real numbers & $pr = 2(q + s)$, then at least one of the equations $x^2 + px + q = 0$, $x^2 + rx + s = 0$ has real roots.

Q.16 If a & b are positive numbers, prove that the equation $\dfrac{1}{x} + \dfrac{1}{x-a} + \dfrac{1}{x+b} = 0$ has two real roots, one between a/3 & 2a/3 and the other between – 2b/3 & – b/3.

Q.17 If the roots of $x^2 - ax + b = 0$ are real & differ by a quantity which is less than c (c > 0), prove that b lies between $(1/4)(a^2 - c^2)$ & $(1/4)a^2$.

Q.18 At what values of 'a' do all the zeroes of the function ,
$f(x) = (a - 2)x^2 + 2ax + a + 3$ lie on the interval (– 2, 1)?

Q.19 If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)} + b = 0$.

Q.20 If p, q, r and s are distinct and different from 2, show that if the points with co-ordinates $\left(\dfrac{p^4}{p-2}, \dfrac{p^3-5}{p-2}\right), \left(\dfrac{q^4}{q-2}, \dfrac{q^3-5}{q-2}\right), \left(\dfrac{r^4}{r-2}, \dfrac{r^3-5}{r-2}\right)$ and $\left(\dfrac{s^4}{s-2}, \dfrac{s^3-5}{s-2}\right)$ are collinear then pqrs = 5 (p + q + r + s) + 2 (pqr + qrs + rsp + spq).

Q.21 The quadratic equation $x^2 + px + q = 0$ where p and q are integers has rational roots. Prove that the roots are all integral.

Q.22 If the quadratic equations $x^2 + bx + ca = 0$ & $x^2 + cx + ab = 0$ have a common root, prove that the equation containing their other root is $x^2 + ax + bc = 0$.

Q.23 If α, β are the roots of $x^2 + px + q = 0$ & $x^{2n} + p^nx^n + q^n = 0$ where n is an even integer, show that α/β, β/α are the roots of $x^n + 1 + (x + 1)^n = 0$.

Q.24 If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2$, $\beta^3 - \beta^2 + \beta + 5$.

Q.25 If each pair of the following three equations $x^2 + p_1x + q_1 = 0$, $x^2 + p_2x + q_2 = 0$ & $x^2 + p_3x + q_3 = 0$ has exactly one root common , prove that; $(p_1 + p_2 + p_3)^2 = 4[p_1p_2 + p_2p_3 + p_3p_1 - q_1 - q_2 - q_3]$.

Q.26 Show that the function $z = 2x^2 + 2xy + y^2 - 2x + 2y + 2$ is not smaller than – 3.

Q.27 Find all real numbers x such that, $\left(x - \dfrac{1}{x}\right)^{\frac{1}{2}} + \left(1 - \dfrac{1}{x}\right)^{\frac{1}{2}} = x$.

Q.28 Find the values of 'a' for which $-3 < [(x^2 + ax - 2)/(x^2 + x + 1)] < 2$ is valid for all real x.

Q.29 Find the minimum value of $\dfrac{\left(x + \dfrac{1}{x}\right)^6 - \left(x^6 + \dfrac{1}{x^6}\right) - 2}{\left(x + \dfrac{1}{x}\right)^3 + x^3 + \dfrac{1}{x^3}}$ for x > 0.

Q.30 Find the product of the real roots of the equation,
$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$$

EXERCISE–2

Q.1 Solve the following where x ∈ R.

(a) $(x - 1)|x^2 - 4x + 3| + 2x^2 + 3x - 5 = 0$

(b) $3|x^2 - 4x + 2| = 5x - 4$

(c) $|x^3 + 1| + x^2 - x - 2 = 0$

(d) $2^{|x+2|} - |2^{x+1} - 1| = 2^{x+1} + 1$

(e) For a ≤ 0, determine all real roots of the equation $x^2 - 2a|x - a| - 3a^2 = 0$.

Q.2 Let a, b, c, d be distinct real numbers and a and b are the roots of quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$ then find the numerical value of $a + b + c + d$.

Q.3 Let $f(x) = ax^2 + bx + c = 0$ has an irrational root r. If $u = \frac{p}{q}$ be any rational number, where a, b, c, p and q are integer. Prove that $\frac{1}{q^2} \le |f(u)|$.

Q.4 Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α & β, where $\alpha < -1$ & $\beta > 1$ then show that $1 + c/a + |b/a| < 0$.

Q.5 If α, β are the roots of the equation, $x^2 - 2x - a^2 + 1 = 0$ and γ, δ are the roots of the equation, $x^2 - 2(a+1)x + a(a-1) = 0$ such that $\alpha, \beta \in (\gamma, \delta)$ then find the values of 'a'.

Q.6 Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (– 32). Find the value of k.

Q.7 If by eleminating x between the equation $x^2 + ax + b = 0$ & $xy + l(x+y) + m = 0$, a quadratic in y is formed whose roots are the same as those of the original quadratic in x. Then prove either $a = 2l$ & $b = m$ or $b + m = al$.

Q.8 If x be real, prove that $\frac{x^2 - 2x\cos\alpha + 1}{x^2 - 2x\cos\beta + 1}$ lies between $\frac{\sin^2\frac{\alpha}{2}}{\sin^2\frac{\beta}{2}}$ and $\frac{\cos^2\frac{\alpha}{2}}{\cos^2\frac{\beta}{2}}$.

Q.9 Solve the equations, $ax^2 + bxy + cy^2 = bx^2 + cxy + ay^2 = d$.

Q.10 Find the values of K so that the quadratic equation $x^2 + 2(K-1)x + K + 5 = 0$ has atleast one positive root.

Q.11 Find the values of 'b' for which the equation $2\log_{\frac{1}{25}}(bx+28) = -\log_5(12 - 4x - x^2)$ has only one solution.

Q.12 Find all the values of the parameter 'a' for which both roots of the quadratic equation $x^2 - ax + 2 = 0$ belong to the interval (0 , 3).

Q.13 Find all the values of the parameters c for which the inequality has at least one solution.
$$1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) \ge \log_2(cx^2 + c).$$

Q.14 Find the values of K for which the equation $x^4 + (1 - 2K)x^2 + K^2 - 1 = 0$;
(a) has no real solution (b) has one real solution

Q.15 Find all the values of the parameter 'a' for which the inequality
$a.9^x + 4(a-1)3^x + a - 1 > 0$ is satisfied for all real values of x.

Q.16 Find the complete set of real values of 'a' for which both roots of the quadratic equation
$(a^2 - 6a + 5)x^2 - \sqrt{a^2 + 2a}\, x + (6a - a^2 - 8) = 0$ lie on either side of the origin.

Q.17 If $g(x) = x^3 + px^2 + qx + r$ where p, q and r are integers. If g (0) and g (–1) are both odd, then prove that the equation g (x) = 0 cannot have three integral roots.

Q.18 Find all numbers p for each of which the least value of the quadratic trinomial $4x^2 - 4px + p^2 - 2p + 2$ on the interval $0 \le x \le 2$ is equal to 3.

Q.19 Let $P(x) = x^2 + bx + c$, where b and c are integer. If P (x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

Q.20 Let x be a positive real. Find the maximum possible value of the expression
$$y = \frac{x^2 + 2 - \sqrt{x^4 + 4}}{x}.$$

EXERCISE–3

Solve the inequality. Where ever base is not given take it as 10.

Q.1 $(\log_2 x)^4 - \left(\log_{\frac{1}{2}}\frac{x^5}{4}\right)^2 - 20\log_2 x + 148 < 0.$

Q.2 $x^{1/\log x} . \log x < 1$

Q.3 $(\log 100x)^2 + (\log 10x)^2 + \log x \le 14$

Q.4 $\log_{1/2}(x+1) > \log_2(2-x)$.

Q.5 $\log_x 2 . \log_{2x} 2 . \log_2 4x > 1$.

Q.6 $\log_{1/5}(2x^2 + 5x + 1) < 0$.

Q.7 $\log_{1/2} x + \log_3 x > 1$.

Q.8 $\log_{x^2}(2+x) < 1$

Q.9 $\log_x \frac{4x+5}{6-5x} < -1$

Q.10 $(\log_{|x+6|} 2) . \log_2(x^2 - x - 2) \ge 1$

Q.11 $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x-5|} \ge 0$

Q.12 $\log_{[(x+6)/3]}[\log_2\{(x-1)/(2+x)\}] > 0$

Q.13 Find out the values of 'a' for which any solution of the inequality, $\frac{\log_3(x^2 - 3x + 7)}{\log_3(3x+2)} < 1$ is also a solution of the inequality, $x^2 + (5 - 2a)x \le 10a$.

Q.14 Solve the inequality $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$.

Q.15 Find the set of values of 'y' for which the inequality, $2\log_{0.5} y^2 - 3 + 2x\log_{0.5} y^2 - x^2 > 0$ is valid for atleast one real value of 'x'.

EXERCISE–4

Q.1 Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie from $\frac{1}{3}$ & 3 for any real x.[JEE '97 , 5]

Q.2 The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is ______. [JEE '97, 2]

Q.3 Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c & d denote the lengths of the sides of the quadrilateral, prove that: $2 \le a^2 + b^2 + c^2 + d^2 \le 4$.

Q.4 In a college of 300 students, every student reads 5 news papers & every news paper is read by 60 students. The number of news papers is:
(A) atleast 30 (B) atmost 20 (C) exactly 25 (D) none of the above

Q.5 If α, β are the roots of the equation $x^2 - bx + c = 0$, then find the equation whose roots are, $(\alpha^2 + \beta^2)(\alpha^3 + \beta^3)$ & $\alpha^5 \beta^3 + \alpha^3 \beta^5 - 2\alpha^4 \beta^4$.

Q.6(i) Let $\alpha + i\beta$; $\alpha, \beta \in R$, be a root of the equation $x^3 + qx + r = 0$; $q, r \in R$. Find a real cubic equation, independent of α & β, whose one root is 2α.

(ii) Find the values of α & β, $0 < \alpha, \beta < \pi/2$, satisfying the following equation,
$\cos\alpha \cos\beta \cos(\alpha + \beta) = -1/8$. [REE '99, 3 + 6]

Q.7(i) In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ & $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \ne 0)$ then :
(A) a + b = c (B) b + c = a (C) a + c = b (D) b = c

(ii) If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then
(A) a < 2 (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) a > 4 [JEE '99, 2 + 2]

Q.8 If α, β are the roots of the equation, $(x - a)(x - b) + c = 0$, find the roots of the equation, $(x - \alpha)(x - \beta) = c$. [REE 2000 (Mains), 3]

Q.9(a) For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:
(A) 1/3 (B) 1 (C) 3 (D) 2/3

(b) If α & β $(\alpha < \beta)$, are the roots of the equation, $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$
(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

(c) If b > a , then the equation, $(x - a)(x - b) - 1 = 0$, has :
(A) both roots in [a, b] (B) both roots in $(-\infty, a)$
(C) both roots in $[b, \infty)$ (D) one root in $(-\infty, a)$ & other in $(b, +\infty)$
[JEE 2000 Screening, 1 + 1 + 1 out of 35]

(d) If α, β are the roots of $ax^2 + bx + c = 0$, $(a \ne 0)$ and $\alpha + \delta$, $\beta + \delta$, are the roots of, $Ax^2 + Bx + C = 0$, $(A \ne 0)$ for some constant δ, then prove that,
$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}.$$ [JEE 2000, Mains, 4 out of 100]

Q.10 The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is [JEE 2001, Screening, 1 out of 35]
(A) 2 (B) 0 (C) 4 (D) 1

Q.11 Let a, b, c be real numbers with $a \ne 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β.
[JEE 2001, Mains, 5 out of 100]

Q.12 The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is
(A) $(-\infty, -2) \cup (2, \infty)$ (B) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
(C) $(-\infty, -1) \cup (1, \infty)$ (D) $(\sqrt{2}, \infty)$ [JEE 2002 (screening), 3]

Q.13 If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. [JEE 2003, Mains-4 out of 60]
[Based on M. R. test]

Q.14(a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then
(A) $p^3 + q^2 - q(3p + 1) = 0$ (B) $p^3 + q^2 + q(1 + 3p) = 0$
(C) $p^3 + q^2 + q(3p - 1) = 0$ (D) $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then
(A) $-5 < a < 2$ (B) $a < -5$ (C) $a > 5$ (D) $2 < a < 5$
[JEE 2004 (Screening)]

Q.15 Find the range of values of *t* for which $2 \sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
[JEE 2005(Mains), 2]

Q.16(a) Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then
(A) $\lambda < \frac{4}{3}$ (B) $\lambda > \frac{5}{3}$ (C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$
[JEE 2006, 3]

(b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then find the value of a + b + c + d. (a, b, c and d are distinct numbers) [JEE 2006, 6]

EXERCISE–5

Part : (A) Only one correct option

1. The roots of the quadratic equation $(a + b - 2c) x^2 - (2a - b - c) x + (a - 2b + c) = 0$ are

(A) $a + b + c$ and $a - b + c$ (B) $\frac{1}{2}$ and $a - 2b + c$

(C) $a - 2b + c$ and $\frac{1}{a+b-c}$ (D) none of these

2. The roots of the equation $2^{x+2} . 3^{\frac{3x}{x-1}} = 9$ are given by

(A) $1 - \log_2 3, 2$ (B) $\log_2 (2/3), 1$ (C) $-2, 2$ (D) $-2, 1 - \frac{\log 3}{\log 2}$

3. Two real numbers α & β are such that $\alpha + \beta = 3$ & $|\alpha - \beta| = 4$, then α & β are the roots of the quadratic equation:

(A) $4x^2 - 12x - 7 = 0$ (B) $4x^2 - 12x + 7 = 0$ (C) $4x^2 - 12x + 25 = 0$ (D) none of these

4. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(A) real roots (B) imaginary roots (C) exactly one root (D) none of these

5. If $e^{\cos x} - e^{-\cos x} = 4$, then the value of cos x is

(A) $\log\left(2+\sqrt{5}\right)$ (B) $-\log\left(2+\sqrt{5}\right)$ (C) $\log\left(-2+\sqrt{5}\right)$ (D) none of these

6. The number of the integer solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :

(A) 1 (B) 2 (C) 3 (D) none

7. If $(x + 1)^2$ is greater than $5x - 1$ & less than $7x - 3$ then the integral value of x is equal to

(A) 1 (B) 2 (C) 3 (D) 4

8. The set of real 'x' satisfying, $\left||x-1|-1\right| \le 1$ is:

(A) [0, 2] (B) [– 1, 3] (C) [– 1, 1] (D) [1, 3]

9. Let $f(x) = x^2 + 4x + 1$. Then

(A) f(x) > 0 for all x (B) f(x) > 1 when $x \ge 0$ (C) $f(x) \ge 1$ when $x \le -4$ (D) f(x) = f(– x) for all x

10. If x is real and $k = \frac{x^2-x+1}{x^2+x+1}$ then:

(A) $\frac{1}{3} \le k \le 3$ (B) $k \ge 5$ (C) $k \le 0$ (D) none

11. If x is real, then $\frac{x^2 - x + c}{x^2 + x + 2c}$ can take all real values if :

(A) $c \in [0, 6]$ (B) $c \in [-6, 0]$ (C) $c \in (-\infty, -6) \cup (0, \infty)$ (D) $c \in (-6, 0)$

12. The solution set of the inequality $\frac{x^4-3x^3+2x^2}{x^2-x-30} \ge 0$ is:

(A) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (B) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

(C) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (D) none of these

13. If $x - y$ and $y - 2x$ are two factors of the expression $x^3 - 3x^2y + \lambda xy^2 + \mu y^3$, then

(A) $\lambda = 11, \mu = -3$ (B) $\lambda = 3, \mu = -11$ (C) $\lambda = \frac{11}{4}, \mu = -\frac{3}{4}$ (D) none of these

14. If α, β are the roots of the equation, $x^2 - 2mx + m^2 - 1 = 0$ then the range of values of m for which $\alpha, \beta \in (-2, 4)$ is:

(A) (– 1, 3) (B) (1, 3) (C) $(\infty, -1) \cup ((3, \infty)$ (D) none

15. If the inequality $(m - 2)x^2 + 8x + m + 4 > 0$ is satisfied for all $x \in R$ then the least integral m is:

(A) 4 (B) 5 (C) 6 (D) none

16. For all $x \in R$, if $mx^2 - 9mx + 5m + 1 > 0$, then m lies in the interval

(A) – (4/61, 0) (B) [0, 4/61) (C) (4/61, 61/4) (D) (– 61/4, 0]

17. Let a > 0, b > 0 & c > 0. Then both the roots of the equation $ax^2 + bx + c = 0$

(A) are real & negative (B) have negative real parts (C) are rational numbers (D) none

18. The value of 'a' for which the sum of the squares of the roots of the equation, $x^2 - (a - 2) x - a - 1 = 0$ assume the least value is:

(A) 0 (B) 1 (C) 2 (D) 3

19. Consider $y = \frac{2x}{1+x^2}$, then the range of expression, $y^2 + y - 2$ is:

(A) $[-1, 1]$ (B) $[0, 1]$ (C) $\left[-9/4, 0\right]$ (D) $\left[-9/4, 1\right]$

20. If both roots of the quadratic equation $x^2 + x + p = 0$ exceed p where $p \in R$ then p must lie in the interval:

(A) $(-\infty, 1)$ (B) $(-\infty, -2)$ (C) $(-\infty, -2) \cup (0, 1/4)$ (D) $(-2, 1)$

21. If a, b, p, q are non–zero real numbers, the two equations, $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have:

(A) no common root (B) one common root if $2a^2 + b^2 = p^2 + q^2$

(C) two common roots if $3pq = 2ab$ (D) two common roots if $3qb = 2ap$

22. If α, β & γ are the roots of the equation, $x^3 - x - 1 = 0$ then, $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value equal to:

(A) zero (B) -1 (C) -7 (D) 1

23. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is:

(A) $(-5, -7)$ (B) $(1, -1)$ (C) $(-1, 1)$ (D) $(5, 7)$

24. If α, β are roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is

(A) $ab\,x^2 - (a + b)\,cx + (a + b)^2 = 0$ (B) $ac\,x^2 - (a + c)\,bx + (a + c)^2 = 0$

(C) $ac\,x^2 + (a + c)\,bx - (a + c)^2 = 0$ (D) none of these

25. If coefficients of the equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and roots of the equation are non-real complex and $a + c < b$, then

(A) $4a + c > 2b$ (B) $4a + c < 2b$ (C) $4a + c = 2b$ (D) none of these

26. The set of possible values of λ for which $x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ has roots, whose sum and product are both less than 1, is

(A) $\left(-1, \frac{5}{2}\right)$ (B) $(1, 4)$ (C) $\left[1, \frac{5}{2}\right]$ (D) $\left(1, \frac{5}{2}\right)$

27. Let conditions C_1 and C_2 be defined as follows : $C_1 : b^2 - 4ac \geq 0$, C_2 : a, –b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if

(A) both C_1 and C_1 are satisfied (B) only C_2 is satisfied

(C) only C_1 is satisfied (D) none of these

Part : (B) May have more than one options correct

28. If a, b are non-zero real numbers, and α, β the roots of $x^2 + ax + b = 0$, then

(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$ (B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$

(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$ (D) $-\alpha, -\beta$ are the roots of $x^2 + ax - b = 0$

29. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is $(a, b, c, d \in R)$

(A) $-d/a$ (B) d/a (C) $(b - a)/a$ (D) $(a - b)/a$

30. If $(x^2 + x + 1) + (x^2 + 2x + 3) + (x^2 + 3x + 5) + \ldots\ldots + (x^2 + 20x + 39) = 4500$, then x is equal to:

(A) 10 (B) -10 (C) 20.5 (D) -20.5

31. $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, then the value of $\sin 2\alpha$ is:

(A) 24/25 (B) $-12/25$ (C) $-24/25$ (D) 20/25

32. If the quadratic equations, $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root then the equation containing their other roots is/are:

(A) $x^2 + a(b + c)x - a^2bc = 0$ (B) $x^2 - a(b + c)x + a^2bc = 0$

(C) $a(b + c)x^2 - (b + c)x + abc = 0$ (D) $a(b + c)x^2 + (b + c)x - abc = 0$

EXERCISE–6

1. Solve the equation, $x(x + 1)(x + 2)(x + 3) = 120$.

2. Solve the following where $x \in R$.

(a) $(x - 1)\left|x^2 - 4x + 3\right| + 2x^2 + 3x - 5 = 0$ (b) $(x + 3)\cdot\left|x + 2\right| + \left|2x + 3\right| + 1 = 0$

(c) $\left|(x + 3)\right|.(x + 1) + \left|2x + 5\right| = 0$ (d) $2^{|x+2|} - \left|2^{x+1} - 1\right| = 2^{x+1} + 1$

3. If 'x' is real, show that, $\frac{(x-1)(x+1)(x+4)(x+6)+25}{7x^2+8x+4} \geq 0$.

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4. Find the value of x which satisfy inequality $\frac{x-2}{x+2} > \frac{2x-3}{4x-1}$.

5. Find the range of the expression $f(x) = \sin^2 x - \sin x + 1 \ \forall\ x \in R$.

6. Find the range of the quadratic expression $f(x) = x^2 - 2x + 3 \ \forall\ x \in [0, 2]$.

7. Prove that the function $y = (x^2 + x + 1)/(x^2 + 1)$ cannot have values greater than 3/2 and values smaller than 1/2 for $\forall\ x \in R$.

8. If x be real, show that $\frac{x^2-2x+9}{x^2+2x+9}$ lies in $\left[\frac{1}{2}, 2\right]$.

9. For what values of k the expression $3x^2 + 2xy + y^2 + 4x + y + k$ can be resolved into two linear factors.

10. Show that one of the roots of the equation, $a x^2 + b x + c = 0$ may be reciprocal of one of the roots of $a_1 x^2 + b_1 x + c_1 = 0$ if $(a a_1 - c c_1)^2 = (b c_1 - a b_1)(b_1 c - a_1 b)$.

11. Let $\alpha + i\beta$; $\alpha, \beta \in R$, be a root of the equation $x^3 + qx + r = 0$; $q, r \in R$. Find a real cubic equation, independent of α and β, whose one root is 2α.

12. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Show that $q^2 - p^2 = (a - c)(b - c)(a + d)(b + d)$.

13. If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ & $\alpha^3 \beta^2 + \alpha^2 \beta^3$.

14. If 'x' is real, find values of 'k' for which, $\left|\frac{x^2+kx+1}{x^2+x+1}\right| < 2$ is valid.

15. Solve the inequality, $\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$.

16. The equations $x^2 - ax + b = 0$ & $x^3 - px^2 + qx = 0$, where $b \neq 0$, $q \neq 0$ have one common root & the second equation has two equal roots. Prove that $2(q + b) = ap$.

17. Find the real values of 'm' for which the equation, $\left(\frac{x}{1+x^2}\right)^2 - (m-3)\left(\frac{x}{1+x^2}\right) + m = 0$ has atleast one real root ?

18. Let a and b be two roots of the equation $x^3 + px^2 + qx + r = 0$ satisfying the relation $ab + 1 = 0$. Prove that $r^2 + pr + q + 1 = 0$.

ANSWER KEY

EXERCISE–1

Q.2 $2x^2 + 2x\cos(A - B) - 2$ **Q.3** 254 **Q.7** $a \in \left(-\infty, -\frac{1}{2}\right)$

Q.8 $x^2 - 4x + 1 = 0$; $\alpha = \tan\left(\frac{\pi}{12}\right)$; $\beta = \tan\left(\frac{5\pi}{12}\right)$ **Q.9** 1 **Q.10** minimum value 3 when x = 1 and p = 0

Q.12 **(a)** (ii) and (iv) ; **(b)** $x^2 - p(p^4 - 5p^2 q + 5q^2)x + p^2 q^2 (p^2 - 4q)(p^2 - q) = 0$

Q.18 $\left(-\infty, -\frac{1}{4}\right) \cup \{2\} \cup (5, 6]$ **Q.24** $x^2 - 3x + 2 = 0$ **Q.27** $x = \frac{\sqrt{5}+1}{2}$ **Q.28** $-2 < a < 1$

Q.29 $y_{min} = 6$ **Q.30** 20

EXERCISE–2

Q.1 **(a)** x = 1; **(b)** x = 2 or 5; **(c)** x = –1 or 1; **(d)** $x \geq -1$ or $x = -3$; **(e)** $x = (1-\sqrt{2})a$ or $(\sqrt{6} - 1)a$

Q.2 30 **Q.5** $a \in \left(-\frac{1}{4}, 1\right)$ **Q.6** k = 86

Q.9 $x^2 = y^2 = d/(a+b+c)$; $x/(c-a) = y/(a-b) = K$ where $K^2 a(a^2+b^2+c^2-ab-bc-ca) = d$

Q 10. $K \leq -1$ **Q 11.** $(-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$ **Q 12.** $2\sqrt{2} \leq a < \frac{11}{3}$

Q.13 (0, 8] **Q 14. (a)** $K < -1$ or $K > 5/4$ **(b)** $K = -1$ **Q 15.** $[1, \infty)$

Q 16. $(-\infty, -2] \cup [0, 1) \cup (2, 4) \cup (5, \infty)$ **Q18.** $a = 1 - \sqrt{2}$ or $5 + \sqrt{10}$

Q.19 P(1) = 4 **Q 20.** $2(\sqrt{2}-1)$ where $x = \sqrt{2}$

EXERCISE–3

Q 1. $x \in \left(\frac{1}{16}, \frac{1}{8}\right) \cup (8, 16)$ **Q 2.** $(0,1) \cup (1, 10^{1/10})$ **Q 3.** $\frac{1}{\sqrt{10}^{9}} \le x \le 10$

Q 4. $-1 < x < \frac{1-\sqrt{5}}{2}$ or $\frac{1+\sqrt{5}}{2} < x < 2$ **Q 5.** $2^{-\sqrt{2}} < x < 2^{-1}$; $1 < x < 2^{\sqrt{2}}$ **Q 6.** $(-\infty, -2.5) \cup (0, \infty)$

Q 7. $0 < x < 3^{1/1-\log 3}$ (where base of log is 2) **Q 8.** $-2<x<-1, -1<x<0, 0<x<1, x>2$ **Q 9.** $\frac{1}{2} < x < 1$

Q 10. $x < -7, -5 < x \le -2, x \ge 4$ **Q 11.** $x \le -\frac{2}{3}$; $\frac{1}{2} \le x \le 2$ **Q12.** $(-6, -5) \cup (-3, -2)$ **Q 13.** $a \ge \frac{5}{2}$

Q 14. $x \in (3, 5-\sqrt{3}) \cup (7, \infty)$ **Q 15.** $(-\infty, -2\sqrt{2}) \cup \left(-\frac{1}{\sqrt{2}}, 0\right) \cup \left(0, \frac{1}{\sqrt{2}}\right) \cup (2\sqrt{2}, \infty)$

EXERCISE–4

Q.2 4 **Q.4** C

Q.5 $x^2 - (x_1 + x_2)x + x_1x_2 = 0$ where $x_1 = (b^2 - 2c)(b^3 - 3cb)$; $x_2 = c^3(b^2 - 4c)$

Q.6 (i) $x^3 + qx - r = 0$, **(ii)** $\alpha = \beta = \pi/3$, **Q.7 (i)** A, **(ii)** A, **Q.8** (a, b) **Q.9 (a)** C, **(b)** B, **(c)** D

Q.10 A **Q.11** $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ **Q.12** B **Q.13** $a > 1$

Q.14 (a) D ; (b) A **Q.15** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ **Q.16** **(a)** A, **(b)** 1210

EXERCISE–5

1. D	**2.** D	**3.** A	**4.** A	**5.** D	**6.** D	**7.** C	**8.** B	**9.** C	**10.** A	**11.** B	
12. B	**13.** C	**14.** A	**15.** B	**16.** B	**17.** B	**18.** B	**19.** C	**20.** B	**21.** A	**22.** C	
23. A	**24.** D	**25.** B	**26.** D	**27.** A	**28.** BC	**29.** AD	**30.** AD	**31.** AC	**32.** BD		

EXERCISE–6

1. {2, – 5} **2.** (a) $x = 1$ (b) $x = (-7-\sqrt{17})/2$
(c) $x = -2, -4, -(1+\sqrt{3})$ (d) $x \ge -1, x = -3$

4. $x \in (-\infty, -2) \cup \left(\frac{1}{4}, 1\right) \cup (4, \infty)$ **5.** $\left[\frac{3}{4}, 3\right]$ **6.** [2, 3] **9.** $k = \frac{11}{8}$ **11.** $x^3 + qx - r = 0$

13. $x^2 - p(p^4 - 5p^2q + 5q^2)x + p^2q^2(p^2 - 4q)(p^2 - q) = 0$ **14.** $k \in (0, 4)$

15. $(-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$ **17.** $\left[\frac{-7}{2}, \frac{5}{6}\right]$

